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On Supersymmetric Effective Theories of Axion

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Abstract

We study effective theories of an axion in spontaneously broken supersymmetric theories. We consider a system where the axion supermultiplet is directly coupled to a supersymmetry breaking sector whereas the standard model sector is communicated with those sectors through loops of messenger fields. The gaugino masses and the axion-gluon coupling necessary for solving the strong CP problem are both obtained by the same effective interaction. We discuss cosmological constraints on this framework.

1 Introduction

The hierarchy problem [1] and the strong CP problem [2] are two big misteries unexplained in the Standard Model of particle physics. Among various candidate solutions proposed so far, supersymmetry (SUSY) and the Peccei-Quinn (PQ) [3] mechanism are promissing ones. Both hypotheses rely on broken symmetries. The construction of the whole picture requires model building efforts of the symmetry breaking sectors and the mediation mechanisms to the Standard Model sector.

The consistency needs to be checked when we combine SUSY and the PQ mechanism. If we are meant to solve the strong CP problem, it is not a good idea to assume a mechanism which causes another CP problem such as the SUSY CP problem. This gives a constraint on the SUSY breaking/mediation sector. Also, by SUSY the axion field is necessarily extended to an axion supermultiplet, which includes a CP-even axion field (saxion). The field value of the saxion (corresponding to the axion decay constant) needs to be stabilized, that requries a coupling to the SUSY breaking sector. The decay constant f_a is constrained by cosmology and astrophysics. The viable region, 10^9 GeV $\lesssim f_a \lesssim 10^{12}$ GeV, is called the axion window [4].

There are numbers of approaches for a supersymmetric axion model in field theories and string theories [5]-[23]. The consistent model building tends to be rather complicated because one needs to carefully glue two symmetry breaking sectors. Especially, field theoretic approaches typically find new particles in both the PQ breaking sector and the SUSY breaking sector. The discussion of the viability is limited to specific models in such cases. On the other hand, the string theory approach tends to predict a too large decay constant [24] for the PQ symmetry breaking, such as the Planck scale, since that is the only scale in the theory unless there is some non-perturbative effects or a large compactification volume. The cosmological difficulty of the saxion (or moduli) field has also been pointed out [10, 25, 26]. The coherent oscillation of the saxion field and its decay produces dangerous particles like gravitino. There are severe constraints from the Big Bang nucleosynthesis (BBN) [27, 28] and also the matter density of the universe [29] if the gravitino is the lightest supersymmetric particle (LSP).

In order to avoid the complexity of the models for the discussion, we study simplified models of SUSY and PQ breaking in this paper. We construct effective theories which cover various microscopic models, although we do make a few assumptions motivated by phenomenology/cosmology and minimality. First, we assume the gauge mediation mechanism for the transmission of the SUSY breaking. This is a good starting point to avoid the SUSY FCNC and CP problem. We assume a direct coupling between the SUSY and the PQ breaking sectors

so that the saxion can obtain a large enough mass to avoid the cosmological difficulty. Finally, we assume that the same messenger fields (or the same effective operator) to communicate the SUSY breaking and PQ breaking to the Standard Model sector.

In the next Section, we will propose a general framework for the simplified model. In Section 3, we will study properties of three representative models classfied by ways to break an R-symmetry. The cosmological constraints are discussed in Section 4. In Section 5, we will give brief comments on the string theory approach. Section 6 will be devoted to the conclusion. In the appendices, we discuss supergravity corrections to the mass spectrum and list relevant computations of the decay rates of heavy fields.

2 Low energy effective Lagrangian

We construct a model where the PQ symmetry is non-linearly realized whereas SUSY is linearly realized. We introduce an axion chiral superfield A and a SUSY breaking chiral superfield X whose F-component obtains a non-vanising vacuum expectation value (vev). In general, X can carry a PQ charge q_X. The superfields X and A transform under the global $U(1)_{\rm PQ}$ symmetry:

$$X \to e^{iq_X\theta_{\rm PQ}}X, \quad A \to A + i\theta_{\rm PQ}. \tag{1}$$

Here $\theta_{\rm PQ}$ is a transformation parameter. Without a loss of generality, we take $q_X \geq 0$.

The effective theory for the PQ breaking sector is given by

$$K_{\rm axion} = f_0^2\left[\frac{1}{2}(A+A^\dagger)^2 + \frac{\hat{C}_3}{3!}(A+A^\dagger)^3 + \cdots\right]. \tag{2}$$

No superpotential cannot be written only with A. The parameter f_0 is the decay constant of the PQ symmetry breaking. We will take the coefficient $\hat{C}_3$ as a dimensionless parameter of $O(1)$. The SUSY breaking sector has the Kähler potential:

$$K_{\rm SB} = \Lambda_0^2\left[X^\dagger X - \frac{a(X^\dagger X)^2}{4} - \frac{(X^\dagger X)^3}{18\hat{x}_0^2} + \cdots\right], \tag{3}$$

where Λ_0 is the typical mass scale of the SUSY breaking sector and the parameter a can be chosen to be $a = \pm 1$ by an rescaling of the field. $\hat{x}_0^2$ is a prameter of order unity. The sign of a controls whether X obtains a vev [30]. The R-symmetry is assumed here so that the SUSY breaking model described below will be justified. In the above Kähler potentials, we took A and X be dimensionless. Note that their values should be limited to $|X| < 1$ and $|A| < 1$ for the validity of the effective theory[1].

[1] In order for X vev not to dominate the axion decay constant, it is required that $f_0/\Lambda_0 > \langle X\rangle$ if $q_X \neq 0$.

The SUSY breaking is described by the Polonyi model [25, 31, 32]. The superpotential is [33, 34, 35]:

$$W_{\rm SB} \;\; = \mu_0^2 (W_0 + \Lambda_0 X e^{-q_X A}). \tag{4}$$

Here the X superfield has an R-charge 2. The first term is a constant which is related to a fine-tuning of the cosmological constant in supergravity, $W_0 \simeq M_{\rm Pl}/\sqrt{3}$. The gravitino mass is $\mu_0^2 W_0 \simeq m_{3/2} M_{\rm Pl}^2$. Here $M_{\rm Pl} = 2.4 \times 10^{18}$ GeV is the reduced Planck mass. Note that $A \to \infty$ can be the SUSY vacuum; in that case a minimum with a finite $\langle A \rangle$ is a metastable SUSY breaking vacuum.

Finally, we write down a coupling between the X and A fields which is necessary for stabilizaing the saxion field:

$$K_{AX} \;\; = \;\; \Lambda_{c,0}^2 \Bigg[\left(c_1 (A + A^\dagger) + \frac{c_2}{2} (A + A^\dagger)^2 \right) X^\dagger X \tag{5}$$

$$-d_1 (A + A^\dagger) \frac{a (X^\dagger X)^2}{4} - e_1 (A + A^\dagger) \frac{(X^\dagger X)^3}{18 \hat{x}_0^2} + \cdots \Bigg]. \tag{6}$$

Again, c_1, c_2, d_1 and e_1 are parameters of order unity. A negative value of c_2 give a mass term for the saxion. The overall scale is set by

$$\Lambda_{c,0} \equiv \min[\Lambda_0, f_0]. \tag{7}$$

The choice of this overall scale is motivated from the discussion below.

For $\Lambda_0 \ll f_0$, the above interaction terms are generated through, for example, a loop of a heavy field in the SUSY breaking sector whose mass carries the PQ charge[2]

$$\Lambda_0^2 X^\dagger X \log |\Lambda_0 e^{qA}|^2. \tag{8}$$

This corresponds to above interaction terms with $\Lambda_{c,0} \sim \Lambda_0$.

For $\Lambda_0 \gg f_0$, on the other hand, the following terms can be generated after integrating out heavy modes in the SUSY breaking sector:

$$S^\dagger S X^\dagger X, \tag{9}$$

where $S = f_0 e^A$. This case corresponds to $\Lambda_{c,0} \sim f_0$. It is possible that the loops of fields in the PQ breaking sector generate $1/f_0^2$ suppressed terms which connect X and A. In that case,

[2] We can consider a O'Raifeartaigh model [36] in the UV scale such like $W_{O'} = \mu^2 e^{-q_X A} X + \frac{\Lambda_0}{4\pi} e^{qA} \phi_1 \phi_2 + \tilde{\lambda} e^{q' A} X \phi_1^2$.

it is natural to assume that the $|X|^4$ term is also generated with the same suppression factor. Therefore, $\Lambda_{c,0} \sim \Lambda_0 \sim f_0$. In any case, one can summarize the overall scale as in Eq. (7).

In summary the effective theory we consider is

$$\begin{aligned} K &= K_{\text{axion}} + K_{\text{SB}} + K_{AX}, \\ W &= W_{\text{SB}}. \end{aligned} \tag{10}$$

We take all the dimensionless parameters to be of $O(1)$.

2.1 Unified origins of the axion-gluon coupling and the gaugino mass

In this subsection, let us consider the origin of an axion coupling to gluons:

$$\int d^2\theta A \text{Tr} W^\alpha W_\alpha, \tag{11}$$

where W^α is the gluon superfield. This term is necessary to solve the strong CP problem. Once F-component of A acquires a vev, the same interaction induces the gluino mass. Since the term $X\text{Tr}W^\alpha W_\alpha$ is forbidden by the R-symmetry, the above interaction term is going to be the leading contribution to the gluino mass.

This unification of the axion coupling and gaugino masses can often be seen in string models (see [12] for the string theoretic QCD axion with intermediate scale decay constant[3]). There, we have a coupling at tree level

$$\frac{1}{2}\int d^2\theta \left(\frac{1}{g_h^2} + A\right) \text{Tr}W^\alpha W_\alpha + c.c. \tag{12}$$

Here $1/g_h^2 = 1/g_0^2 - i\vartheta/8\pi^2$. Now $\text{Im}(A)$ can be identified with (a linear combination of) the QCD axion and $(g_0^2/2)F^A$ is a gaugino mass, which can be comparable to or less than the gravitino mass.

In gauge mediation models [39, 40, 41, 42], the above coupling can be obtained after integrating out messenger fields whose mass carries a PQ charge[4],

$$W_{\text{Mess}} = M_0 e^{-q_{\Psi\bar{\Psi}}A}\Psi\bar{\Psi}, \tag{13}$$

where $q_{\Psi\bar{\Psi}} \equiv PQ(\Psi) + PQ(\bar{\Psi})$ and Ψ and $\bar{\Psi}$ are messenger fields. After integrating out the messenger fields, we have the axion coupling to gauge multiplets,

$$\frac{1}{2}\int d^2\theta \left(\frac{1}{g_h^2} - q_{\Psi\bar{\Psi}}N_{\Psi\bar{\Psi}}\frac{A}{8\pi^2}\right) \text{Tr}W^\alpha W_\alpha + c.c. \tag{14}$$

[3] See also topics related with LARGE volume scenario [37, 38, 21].

[4] Even though we have an additional R-breaking operator $W = \lambda'\Lambda X e^{-(q_X + q_{\Psi\bar{\Psi}})A}\Psi\bar{\Psi}$, it is irrelevant to mass spectra, so far as the following conditions are satisfied: $M_0 > \lambda'\Lambda X e^{-q_X A}$ and $M_0 F^A > \lambda'\Lambda F^X e^{-q_X A}$.

This is a hadronic axion model [43]. Here $N_{\Psi\bar{\Psi}}$ is the number of messenger multiplets[5]. Im(A) is the QCD axion and $q_{\Psi\bar{\Psi}}N_{\Psi\bar{\Psi}}(\alpha/4\pi)F^A$ is the gaugino mass, which can be larger than the gravitino mass.

2.2 Standard Form

We now define a particular basis to proceed the discussion. First, one can eliminate A in the superpotential by a redefinition of X by

$$X \to e^{q_X A} X. \tag{15}$$

Next, we can also eliminate the $(A + A^\dagger)X^\dagger X$ term by an appropriate shift of A. The vev $\langle A \rangle$ in this basis is vanishing up to small R-breaking effects we discuss later. With new definitions of parameters, the superpotential and the Kähler potential are given by

$$W = \mu^2(W_0 + \Lambda X), \tag{16}$$

$$K = K_{\rm axion} + K_{\rm SB} + K_{AX}, \tag{17}$$

where

$$K_{\rm axion} = f^2 \left[\frac{1}{2}(A + A^\dagger)^2 + \frac{C_3}{3!}(A + A^\dagger)^3 + \cdots \right], \tag{18}$$

$$K_{\rm SB} = \Lambda^2 \left[X^\dagger X - \frac{a(X^\dagger X)^2}{4} - \frac{(X^\dagger X)^3}{18x_0^2} + \cdots \right], \tag{19}$$

$$K_{AX} = \Lambda_c^2 \left[-\frac{\tilde{c}}{2}(A + A^\dagger)^2 X^\dagger X - \tilde{d}(A + A^\dagger)\frac{a(X^\dagger X)^2}{4} - \tilde{e}(A + A^\dagger)\frac{(X^\dagger X)^3}{18x_0^2} + \cdots \right], \tag{20}$$

and

$$\Lambda_c = \min[\Lambda, f]. \tag{21}$$

The gauge kinetic term is

$$\int d^2\theta \left(\frac{1}{g_h^2} + \frac{k}{8\pi^2}A \right) W^\alpha W_\alpha. \tag{22}$$

We introduced a parameter k that depends on the messenger mechanism.

[5] Once we require $q_{\Psi\bar{\Psi}}N_{\Psi\bar{\Psi}}$ should be an integer, a value that $q_{\Psi\bar{\Psi}}N_{\Psi\bar{\Psi}} = 1$ is desirable considering the domain wall problem [44, 45]

One needs to check if the shift we performed is within the allowed range in the effective theory. In terms of the original parameters, the shift is given by

$$\langle A_0 \rangle = -\frac{c_1}{2c_2} - \frac{1}{2}\left(\frac{1}{q_X} - \xi\right), \tag{23}$$

where

$$\xi = \sqrt{\left(\frac{c_1}{c_2}\right)^2 + \frac{1}{q_X^2} - \frac{2}{c_2}\left(\frac{\Lambda_0}{\Lambda_{c,0}}\right)^2}. \tag{24}$$

For $|\langle A_0 \rangle| \lesssim 1$, we need $q_X = 0$ for $\Lambda \gg f$.

3 R-breaking Models

The scalar potential can simply be calculated from (16) and (17):

$$V = K^{X^\dagger X} |\partial_X W|^2. \tag{25}$$

Recall that a small R-symmetry breaking effect is required to produce gaugino mass; a vev of F^A is vanishing without R-breaking effects.

We consider three types of R-symmetry breaking in the following. The first one (Model 1) is a model with spontaneous R-symmetry breaking, that can be achieved by taking the a parameter to be -1. The second model (Model 2) is to add a small explicit R-breaking term in the Kähler potential:

$$\Delta K = \epsilon_K \Lambda^2 (X + X^\dagger)(A + A^\dagger). \tag{26}$$

Finally, the third model (Model 3) is to add a small explicit R-breaking term in the superpotential[6]:

$$\Delta W = \epsilon \mu^2 \Lambda \cdot \frac{X^2}{2}. \tag{27}$$

We assume $a = 1$ in Model 2 and 3.

We discuss in the following the mass spectrum in each model.

[6] ΔK can be, for example, generated from an additional R-breaking messenger term $W = \lambda' \Lambda X e^{-(q_X + q_{\Psi\bar{\Psi}})A} \Psi \bar{\Psi}$ [46]. On the other hand, ΔW can originate from a non-renormalizable superpotential $W = \frac{\Lambda^2}{m} s^2 X^2$, where $s = f e^{-q_X A}$.

3.1 Model 0

Before going to the discussion of the R-breaking models listed above, we comment on the effect of gravity mediation. Since the R-symmetry is explicitly broken in the supergravity Lagrangian, F^A is induced and the gauginos obtain masses of $O(m_{3/2})$.

The axino (the fermionic component of A) and the saxion (A_R) can also obtain masses. The axino mass is of $O(m_{3/2})$ and

$$m_{A_R} = \frac{\Lambda_c}{\Lambda}\sqrt{6\tilde{c}}\frac{m_{3/2}M_{\rm Pl}}{f}. \tag{28}$$

The mass of the scalar component of X is

$$m_X = \sqrt{3}\frac{m_{3/2}M_{\rm Pl}}{\Lambda}. \tag{29}$$

This is the minimal model of the supersymmetric axion. However, the gravity mediation potentially has problems of large CP violation which we are trying to solve. Therefore, we consider cases with $m_{3/2} \ll 100$ GeV where the supergravity effects is subdominant.

3.2 Model 1: spontaneous R-breaking model with $a = -1,\ \Delta K = 0,\ \Delta W = 0$

For $a = -1$, $X = 0$ is unstable and the R symmetry is spontaneously broken by its vev. The potential is stabilized through the Kähler potential $K \sim -(X^\dagger X)^3$. The vevs of X and A are

$$\begin{aligned}\langle X\rangle &= x_0 + \frac{(\tilde{d}-\tilde{e})(2\tilde{d}-\tilde{e})}{4\tilde{c}}\left(\frac{\Lambda_c}{\Lambda}\right)^2 x_0^3,\\ \langle A\rangle &= \frac{2\tilde{d}-\tilde{e}}{4\tilde{c}}x_0^2.\end{aligned} \tag{30}$$

At the vacuum, the masses for the CP-even scalar fields are obtained to be

$$m_{X_R}^2 = \frac{2\mu^4}{\Lambda^2},\quad m_{A_R}^2 = \frac{2\tilde{c}\mu^4}{f^2}\left(\frac{\Lambda_c}{\Lambda}\right)^2. \tag{31}$$

Here $X = (x_0 + X_R)e^{iX_I}$, $A = A_R + iA_I$ and the above expressions are correct up to of $O(x_0^4)$. The parameter x_0 should satisfy a condition[7]

$$x_0 \lesssim 1, \tag{32}$$

for the effective theory to be valid. Since the R-symmetry is spontaneously broken, there is a nearly massless R-axion X_I. The X_I field and the goldstino (the fermionic component of X)

[7] $\langle X\rangle \simeq x_0 < 1$ also satisfies the condition to obtain the stable vacuum. This is similar to Model 3.

acquire masses through a supergravity correction:

$$m_{X_I}^2 = \frac{2\mu^4}{\sqrt{3}M_{\rm Pl}\Lambda x_0} = 2\sqrt{3}m_{3/2}^2\left(\frac{M_{\rm Pl}}{\Lambda x_0}\right) \tag{33}$$

up to corrections suppressed by the Planck scale and of $O(x_0^2)$. The axion A_I remains massless at this stage.

The axino $\tilde{a}$, which is a fermionic part of A, obtains a mass via a Kähler term:

$$K = -\Lambda_c^2 \cdot \frac{\tilde{c}}{2}(A + A^\dagger)^2 X^\dagger X. \tag{34}$$

It is given by

$$m_{\tilde{a}} = -\frac{\tilde{c}\mu^2\Lambda x_0}{f^2}\left(\frac{\Lambda_c}{\Lambda}\right)^2 \tag{35}$$

and a fermionic part of X, ψ_X, is the goldstino which is absorbed into gravitino. Finally, the F component vev for the axion is

$$F^A = -\frac{(3\tilde{d} - 2\tilde{e})}{6}\frac{\mu^2\Lambda}{f^2}\left(\frac{\Lambda_c}{\Lambda}\right)^2 x_0^3. \tag{36}$$

This gives the gaugino masses, $M_{1/2} = (k\alpha/4\pi)F^A$.

3.3 Model 2: explicit R-breaking model with $a = 1,\ \Delta K \neq 0,\ \Delta W = 0$

This model is R-symmetric if we neglect ϵ_K. Thus we have the following scalar mass spectra up to $O(\epsilon_K^2)$:

$$m_{X_R}^2 = m_{X_I}^2 = \frac{\mu^4}{\Lambda^2}, \tag{37}$$

$$m_{A_R}^2 = \frac{2\tilde{c}\mu^4}{f^2}\left(\frac{\Lambda_c}{\Lambda}\right)^2. \tag{38}$$

Here we denoted as $X = X_R + iX_I$. Of course, the axion A_I and goldstino ψ_X remain massless at this stage.

The shift of the vevs at the leading order in the ϵ_K expansion are[8]

$$\delta X = \epsilon_K^3 C_3 \frac{\Lambda^4}{f^4}, \tag{39}$$

$$\delta A = \epsilon_K^2 C_3 \frac{\Lambda^2}{2\tilde{c}f^2}\left(\frac{\Lambda_c}{\Lambda}\right)^{-2}. \tag{40}$$

[8] For small ϵ_K, the SUGRA correction to δX may have the same order of the magnitude as the following result.

The mass mixing between X_R and A_R is generated by ϵ_K.

$$m^2_{XA} = \frac{\epsilon_K}{2}\frac{\mu^4\Lambda}{f^3}\left(-6\tilde{c}\left(\frac{\Lambda_c}{\Lambda}\right)^2 + \frac{f^2}{\Lambda^2}\right). \tag{41}$$

In order not for the vev shifts to be larger than $O(1)$ and also not to destabilize the vacuum by the mass mixing, we obtain a condition

$$\epsilon_K < \left(\frac{\Lambda_c}{\Lambda}\right)^2. \tag{42}$$

The F-component vev of A is:

$$F^A = \epsilon_K \frac{\mu^2\Lambda}{f^2}. \tag{43}$$

This induces the gaugino masses and the axino mass through a Kähler term $(f^2C_3/3!)(A+A^\dagger)^3$:

$$m_{\tilde{a}} = C_3 F^A. \tag{44}$$

3.4 Model 3: Explicit R-breaking model with $a = 1,\ \Delta K = 0,\ \Delta W \neq 0$

In this model, scalar and goldstino mass spectra are the same as Model 2 up to $O(\epsilon^2)$. Here the vev shift of X is

$$\delta X = -\epsilon, \quad \delta A = 0. \tag{45}$$

Therefore, we have a condition to make our effective theory valid:

$$\epsilon \lesssim 1, \tag{46}$$

for validity of the effective theory. Then the axino obtains a mass via $K = \Lambda_c^2\tilde{c}(A + A^\dagger)^2X^\dagger X$ as in Model 1:

$$m_{\tilde{a}} = -\tilde{c}\epsilon \cdot \frac{\mu^2\Lambda}{f^2}\left(\frac{\Lambda_c}{\Lambda}\right)^2. \tag{47}$$

The F-component of A is

$$F^A = \frac{\epsilon^3\tilde{d}}{2}\frac{\mu^2\Lambda}{f^2}\left(\frac{\Lambda_c}{\Lambda}\right)^2. \tag{48}$$

3.5 Summary of mass spectra and constraints on parameters in three models

Here we summarize the mass spectra of three models in Table 1. Recall we have several parameters:

$$\tilde{c},\ \tilde{d},\ \tilde{e} \lesssim O(1), \quad C_3 = O(1).$$

There are also constraints on R-breaking parameters in each model:

$$\text{Model 1}: x_0 \lesssim 1, \tag{49}$$

$$\text{Model 2}: \ \epsilon_K \lesssim \left(\frac{\Lambda_c}{\Lambda}\right)^2, \tag{50}$$

$$\text{Model 3}: \ \epsilon \lesssim 1. \tag{51}$$

	Model 1	Model 2	Model 3
$m^2_{X_R}$	$\left(\sqrt{2}F^X\right)^2$	$\left(F^X\right)^2$	$\left(F^X\right)^2$
$m^2_{X_I}$	$2\sqrt{3}m^2_{3/2}\left(\frac{M_{\rm Pl}}{x_0\Lambda}\right)$	$\left(F^X\right)^2$	$\left(F^X\right)^2$
$m^2_{A_R}$	$\left(\frac{\Lambda_c}{f}\sqrt{2\tilde{c}}F^X\right)^2$	$\left(\frac{\Lambda_c}{f}\sqrt{2\tilde{c}}F^X\right)^2$	$\left(\frac{\Lambda_c}{f}\sqrt{2\tilde{c}}F^X\right)^2$
$m_{\tilde{a}}$	$x_0\tilde{c}\left(\frac{\Lambda_c}{f}\right)^2 F^X$	$C_3 F_A$	$\epsilon\tilde{c}\left(\frac{\Lambda_c}{f}\right)^2 F^X$
F_A	$-\frac{(2\tilde{e}-3\tilde{d})x_0^3}{6}\frac{\Lambda_c^2}{f^2}F^X$	$-\epsilon_K \cdot \frac{\Lambda^2}{f^2}F^X$	$-\frac{\tilde{d}\epsilon^3}{2}\frac{\Lambda_c^2}{f^2}F^X$

Table 1: A table of mass spectra in three models for a gauge mediation with light gravitino. Here $\Lambda F^X \simeq -\mu^2 \simeq -\sqrt{3}m_{3/2}M_{\rm Pl}$ and $\Lambda_c = \min[\Lambda, f]$. The soft mass in the visible sector are given by $M_{1/2} \simeq k\frac{\alpha}{4\pi}F^A$ and $m_0 \simeq \sqrt{k}\frac{\alpha}{4\pi}F^A$.

The masses in the table should be smaller than the cut-off scales, $\Lambda/4\pi$ in the SUSY breaking sector or f in the PQ breaking sector. Therefore, we have a constraint:

$$\Lambda > \sqrt{4\pi}\mu. \tag{52}$$

4 Cosmological constraints

In this section, we discuss cosmological constraints on three models defined above. We consider decays of the coherent oscillations of the saxion and X_R (the Polonyi field) and discuss constraints

from the BBN and the matter energy density of the universe. We assume that the LSP is the gravitino and the next-to-lightest supersymmetric particle (NLSP) is the bino. The bino NLSP gives more stringent constraint than a case for stau NLSP.

In the following discussion, we fix the PQ breaking scale f to be 10^{10} GeV, bino mass $m_{\tilde{B}}$ to be 100 GeV and all dimensionless parameters (except for x_0, ϵ_K and ϵ) to be of $O(1)$. Once we fix f and the gaugino mass, the parameters we have are Λ and the gravitino mass (or equivalently the R-breaking parameters in each model (x_0, ϵ_K, or ϵ)) aside from the $O(1)$ parameters. See Figure 1–3 for the parameter regions constrained by the validity of the effective theory.

In general, light scalar fields such as X_R and A_R are problematic for cosmology since their coherent oscillations and late-time decays would produce too large entropy and also produce unwanted particles. In the models we are discussing, the Polonyi and the saxion can be much heavier than the Standard Model particles due to the direct coupling to the SUSY breaking sector. This situation helps to cure the problem. We consider here the constraints on the model parameters from the successfull BBN and the matter density of the universe [10, 25, 26].

We define several important temperatures for discussion. The decay temperature T_d^ϕ is the one at which a scalar condensate $\phi(= X_R, X_I, A_R)$ decays in the radiation-dominated universe. The domination temperature $T_{\rm dom}^\phi$ is the one at which ϕ dominates over the energy density of the universe [47, 48, 49]:

$$T_d^\phi \equiv \left(\frac{90}{\pi^2 g_*(T_d^\phi)}\right)^{1/4} \sqrt{\Gamma_\phi M_{\rm Pl}}, \tag{53}$$

$$T_{\rm dom}^\phi \equiv \min[T_R,\ T_{\rm osc}^\phi] \left(\frac{\Delta\phi}{\sqrt{3}M_{\rm Pl}}\right)^2. \tag{54}$$

Here $g_*(T_d^\phi)$ is the effective number of light particles at T_d^ϕ, Γ_ϕ is total decay width of ϕ. T_R is a reheating temperature by an inflaton decay, $T_{\rm osc}^\phi \simeq 0.3\sqrt{m_\phi M_{\rm Pl}}$ is a temperature when ϕ starts to oscillate and $\Delta\phi$ is the initial amplitude. These will be important for our discussion below since ϕ does not dominate over the universe if $T_d^\phi > T_{\rm dom}^\phi$.

4.1 Model 1

We first calculate the decay widths of particles in Model 1.

- X_R (Polonyi) and A_R (saxion) decay

Through the interactions $K = \Lambda^2[X^\dagger X - (X^\dagger X)^2/4]$, the Polonyi field decays into R-axions or gravitinos. The total decay width is given by

$$\Gamma_{X_R} \simeq \Gamma(X_R \to X_I X_I) + \Gamma(X_R \to \psi_{3/2}\psi_{3/2}) \tag{55}$$

$$\simeq \frac{1}{32\pi}\frac{m_{X_R}^3}{x_0^2\Lambda^2} + \frac{1}{96\pi}\frac{m_{X_R}^5}{m_{3/2}^2 M_{\rm Pl}^2}. \tag{56}$$

The vev x_0 can be expressed in terms of the R-axion mass as

$$x_0\Lambda = 2\sqrt{3}M_{\rm Pl}\left(\frac{m_{3/2}}{m_{X_I}}\right)^2. \tag{57}$$

For the saxion, the main decay modes are $A_R \to aa$ and $A_R \to \psi_{3/2} + \tilde{a}$, which originate from $K = f^2(C_3/3)(A + A^\dagger)^3 - \Lambda_c^2(\tilde{c}/2)(A + A^\dagger)^2 X^\dagger X$. The decay width is

$$\Gamma_{A_R} \simeq \Gamma(A_R \to aa) + \Gamma(A_R \to \psi_{3/2} + \tilde{a})$$

$$\simeq \frac{C_3^2}{32\pi}\frac{m_{A_R}^3}{f^2} + \frac{1}{96\pi}\frac{m_{A_R}^5}{m_{3/2}^2 M_{\rm Pl}^2}. \tag{58}$$

The axino ($\tilde{a}$) produced by the saxion decay subsequently decays into gravitinos or the Standard Model particles with the decay width:

$$\Gamma_{\tilde{a}} \simeq \Gamma(\tilde{a} \to \psi_{3/2} + a) + \Gamma(\tilde{a} \to \lambda + g) \tag{59}$$

$$\simeq \frac{1}{96\pi}\frac{m_{\tilde{a}}^5}{m_{3/2}^2 M_{\rm Pl}^2} + N_g \frac{\alpha^2}{256\pi^3}\frac{m_{\tilde{a}}^3}{f_a^2}, \tag{60}$$

where N_g (=12) is the number of the decay modes[9].

Just by looking at the main decay modes, one can see that it is problematic if the coherent oscillations of the Polonyi field or the saxion field dominate over the energy density of the universe[10]. The decay products are stable (or long-lived) particles such as gravitinos and axions which contributes to the matter energy density of the universe. The overproduction of those particles needs to be avoided for viable cosmology.

For of $O(M_{\rm Pl})$ initial amplitudes of the Polonyi and the saxion, the condition for the matter density $\Omega_{\rm matter} \sim 0.2$ requires a low enough reheating temperature after inflation such as $T_R \lesssim 1$ MeV. Such a low reheating temperature may be barely consistent with the BBN. However, we do not consider this case since $O(M_{\rm Pl})$ field values are beyond the validity of the effective

[9] There is also a decay mode $\tilde{a} \to \psi_{3/2} + X_I$, which can be the main decay mode in a narrow region in the parameter space.

[10] If the saxion $s = fe^A$ is captured at the origin $s = 0$ during the inflation, the thermal effect via messenger fields becomes relevant to the saxion potential. For such a case, the saxion can dominate the energy density of the universe [17].

theory. One can consider possibilities that the initial amplitudes are at $\Delta X_R = O(\Lambda)$ [51] or $\Delta A_R = O(f)$ [20], since Λ and f are the unique (cut-off) scales for X and A_R, respectively[11]. For such a case, the Polonyi (the saxion) decays before dominating the universe, with the parameter region that we are considering, since conditions $T^X_{\rm dom} < T^X_d$ and $T^{A_R}_{\rm dom} < T^{A_R}_d$ are always satisfied. Hereafter we consider such a case to search for viable parameter regions.

- X_I (R-axion) decays

The R-axion decays into two gauginos if it is kinematically allowed. The interaction term in the Lagrangian is [49]

$$- i\frac{X_I}{\sqrt{2}x_0\Lambda}\frac{M_{1/2}}{2}\lambda\lambda + c.c.. \tag{61}$$

Here λ is the MSSM gauginos. The total decay width is

$$\begin{aligned}\Gamma_{X_I} &\simeq \Gamma(X_I \to \lambda\lambda) &(62)\\ &\simeq \frac{N_g}{32\pi}m_{X_I}\left(\frac{M_{1/2}}{x_0\Lambda}\right)^2\left(1-\frac{4M_{1/2}^2}{m_{X_I}^2}\right)^{1/2}. &(63)\end{aligned}$$

For $m_{X_I} < 2M_{1/2}$, the channel $X_I \to \lambda\lambda$ is closed, then $X_I \to b\bar{b}$ is the main decay mode through the mixing between X_I and the CP-odd Higgs boson (A) in the MSSM[12]. The mixing is obtained through the $B\mu$-term. Although we need a concrete model to generate $B\mu$-term to discuss that interaction, there is always a contribution from a one-loop diagram with the gaugino mass insertion [53, 54], even if we have $B\mu = 0$ at the messenger scale [55]. If that is the donimant contribution, there is an interaction term to mix X_I and A:

$$\frac{i}{2\sqrt{2}}\frac{m_A^2\sin 2\beta}{x_0\Lambda}X_I H_u H_d + c.c., \tag{64}$$

where we have used a relation $B\mu = m_A^2 \sin 2\beta/2$. From this interaction and Yukawa coupling, the decay width is found to be [49]:

$$\Gamma_{X_I} \simeq \Gamma(X_I \to b\bar{b}) \simeq \frac{3m_{X_I}}{16\pi}\left(\frac{m_A^2\sin^2\beta}{x_0\Lambda}\frac{m_b}{m_A^2-m_{X_I}^2}\right)^2\sqrt{1-\frac{4m_b^2}{m_{X_I}^2}}. \tag{65}$$

For further smaller m_{X_I}, $X_I \to \tau\bar{\tau}$ can be the main decay mode with large $\tan\beta$; m_b should be replaced with m_τ. In the parameter region that we are interested, $T_d^{X_I} \gtrsim 1$ MeV[13].

[11] A possiblity $\Delta A_R = O(\sqrt{f_a M_{\rm GUT}})$ was also considered in Ref. [52].

[12] We neglected a decay mode $X_I \to t\bar{t}$ via the similar interaction, since we assumed $m_t > m_{\tilde{B}} = 100$ GeV. On the other hand, note also that so long as $\tan\beta > \sqrt{m_t/m_b} \approx 6.4$ this mode is suppressed, compared to $X_I \to b\bar{b}$. In a whole computation, we assumed m_A and m_{X_I} do not degenerate.

[13] If the decay of X_I is too late, it can influence the BBN. This excludes a small parameter region around $\Lambda \sim 10^{13}$ GeV and $m_{3/2} \sim 40$ MeV in Figure 1 [27].

On the other hand, because $x_0\Lambda$ is a normalization of the R-axion, its initial amplitude is at most on the order of $x_0\Lambda$. The temperature at which oscillating R-axion dominates the universe is given by

$$\begin{aligned} T_{\rm dom}^{X_I} &= \frac{1}{4}\min[T_R,\ T_{\rm osc}^{X_I}]\left(\frac{m_{3/2}}{m_{X_I}}\right)^4\left(\frac{\Delta X_I}{x_0\Lambda}\right)^2 \\ &\lesssim 0.16\text{MeV}\left(\frac{\min[T_R,\ T_{\rm osc}^{X_I}]}{10^{10}\text{GeV}}\right)\left(\frac{m_{3/2}/m_{X_I}}{5\times10^{-4}}\right)^4\left(\frac{\Delta X_I}{x_0\Lambda}\right)^2. \end{aligned}$$

Here $T_{\rm osc}^{X_I} \simeq 0.3\sqrt{m_{X_I}M_{\rm Pl}} \simeq 1.6\times10^{10}\text{GeV}\,(m_{X_I}/1.2\text{TeV})^{1/2}$ is a temperature where the R-axion starts to oscillate. The relation $m_{3/2}/m_{X_I} \lesssim 5\times10^{-4}$ holds for $m_{3/2} \lesssim 40$ MeV which we need from the BBN constraint we discuss later. Then we obtain $T_d^{X_I} \gg T_{\rm dom}^{X_I}$ in our model; the R-axion does not dominate over the universe.

- Bino NLSP and BBN constraint

We have seen that the Polonyi field, the saxion, or R-axion do not dominate over the energy density of the universe provided that the initial amplitudes of the Polonyi field and the saxion are of $O(\Lambda)$ and of $O(f)$, respectively. Then we need to consider the bino abundance since it is a long-lived NLSP, which can disturb the BBN. The thermal abundance of binos is given by [56]

$$Y_{\tilde{B}}^{\rm th} = \frac{n_{\tilde{B}}}{s} = 4\times10^{-12}\times\left(\frac{m_{\tilde{B}}}{100\text{GeV}}\right), \tag{66}$$

below the freeze-out temperature $T_f^{\tilde{B}} \sim m_{\tilde{B}}/30$.

Since the binos decay into gravitino at a later time, there is constraint from the BBN on the lifetime as mentioned above. With the yield of Eq. (66), this can be translated to the bound on the gravitino mass which is [28]

$$m_{3/2} \lesssim 40\ \text{MeV} \quad \text{for } m_{\tilde{B}} = 100\ \text{GeV}. \tag{67}$$

For a larger bino mass or for the stau NLSP case, the constraint is relaxed, such as $m_{3/2} \lesssim 1$ GeV. See Figure 1 for the allowed region.

There are contributions to the bino density from the decays of the heavy field such as X_R, A_R and $\tilde{a}$. Since they are heavy enough, the binos produced by those decays are thermalized and thus such contributions are already taken into account in Eq. (66). The decays of X_I can be later than the freezing-out of the bino-pair annihilation. This non-thermal contribution is smaller than the thermal piece when

$$T_R \lesssim 10^6\ \text{GeV}, \tag{68}$$

which is required later by the constraint from the gravitino abundance.

In summary in Model 1 the gravitino mass is constrained to be

$$10^{-4}\ \text{GeV} \lesssim m_{3/2} \lesssim 40\ \text{MeV}, \tag{69}$$

and the cut-off scale (or dynamical scale) of the SUSY breaking sector is

$$10^{8}\ \text{GeV} \lesssim \Lambda \lesssim 10^{13}\ \text{GeV}, \tag{70}$$

for $f = 10^{10}$ GeV.

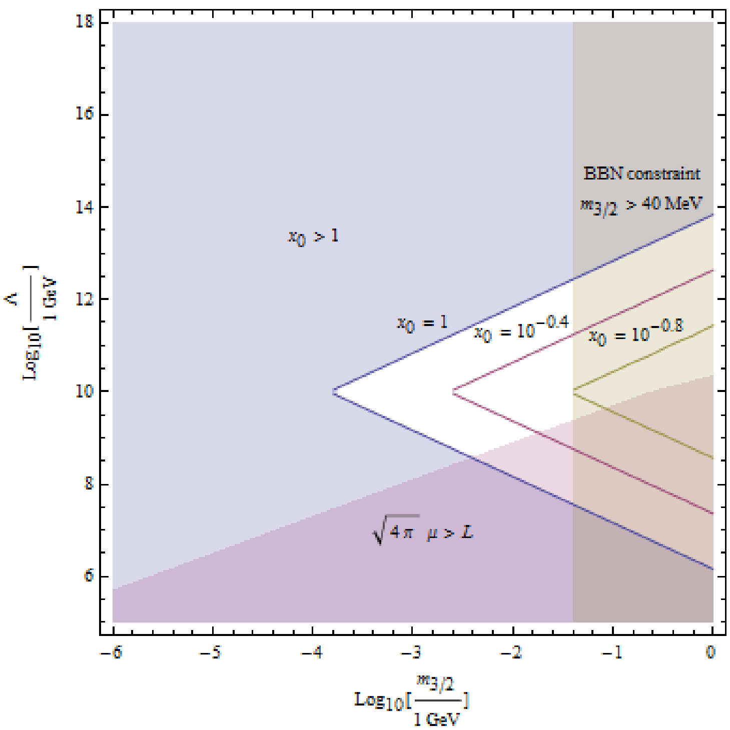

Figure 1: A figure of allowed parameter region for $M_{1/2} = 100$ GeV. Recall that we have constraint that should satisfy $\sqrt{4\pi}\mu \lesssim L$, $x_0 \lesssim 1$ and $m_{3/2} \lesssim 40$ MeV. Here $L = (\sqrt{2}x_0)^{1/2}\Lambda \lesssim \Lambda$.

- Candidates of cold dark matter

The thermal contribution to the gravitino density $\Omega_{3/2}^{\text{th}}$ is [57]:

$$\Omega_{3/2}^{\text{th}} = 0.2\left(\frac{T_R}{3.1\times 10^{6}\text{GeV}}\right)\left(\frac{30\text{MeV}}{m_{3/2}}\right)\left(\frac{M_{\tilde{g}}}{600\text{GeV}}\right)^2. \tag{71}$$

Here $M_{\tilde{g}}$ is gluino mass at TeV scale. Once one fixes gluino mass, the gravitino by thermal scattering can be dark matter of the universe for $T_R \lesssim O(10^6)$ GeV $\times \big(m_{3/2}/O(10) \text{ MeV}\big)$.

Note that we also have non-thermal component to the gravitino density. The important contribution is from the Polonyi decay:

$$Y_{3/2}^{X_R} \simeq \frac{3T_{\rm dom}^{X_R}}{2m_{X_R}} B_{3/2}^{X_R}. \tag{72}$$

Here $B_{3/2}^{X_R}$ is a branching ratio of a decay mode $X_R \to 2\psi_{3/2}$. Then we obtain

$$\Omega_{3/2}^{\rm NT} \simeq 0.23 \left(\frac{\Lambda}{10^{13}\text{GeV}}\right)^3 \left(\frac{T_R}{10^6\text{GeV}}\right) B_{3/2}^{X_R}. \tag{73}$$

Here $B_{3/2}^{X_R} = O(1)$. For $\Lambda \simeq 10^{13}$ GeV and $m_{3/2} \simeq 40$ MeV (near the upper right boundary of the Figure 1), the non-thermal component can be as important as the thermal one. Considering the BBN constraints, the contribution to the gravitino density from the decays of binos, saxions, and the axinos[14] is much smaller than this contribution for $T_R \lesssim O(10^6)$ GeV and $m_{3/2} \lesssim 40$ MeV. This is because of low $T_{\rm dom}$ and small branching ratios into gravitinos.

The axion is also a candidate for dark matter [45]. The abundance is given by

$$\Omega_a \simeq 1.4 \left(\frac{\Theta_{\rm mis}}{\pi}\right)^2 \left(\frac{f_a}{10^{12}\text{GeV}}\right)^{7/6}, \tag{74}$$

where $\Theta_{\rm mis}$ is misalignment angle of the axion.

4.2 Model 2 and Model 3

Essentially, the discussion is parallel to Model 1. A difference is that the R-axion now has a similar mass to the Polonyi field, and thus we do not need to consider it separately As in Model 1, the domination of the Polonyi and the saxion fields would produce too much gravitinos, and

[14] Recently, a paper [68] discussed the constraint on the reheating temperature obtained from the matter energy density of the thermally produced axinos [69] or gravitinos from their decays. In our model, the gravitino abundance from the thermally produced axino is given by

$$\Omega_{3/2}^{\tilde{a}} \simeq 0.2 \left(\frac{T_R}{8.7\times 10^5\text{GeV}}\right)\left(\frac{m_{3/2}}{30\text{MeV}}\right)\left(\frac{10^{12}\text{GeV}}{f_a}\right)^2 B_{3/2}^{\tilde{a}}.$$

Here we have replaced $M_{\tilde{g}}$ with $\sqrt{6}\alpha_3 m_{3/2} M_{\rm Pl}/(4\pi f_a)$ and used $g_3 \simeq 1$ in eq.(71), and $B_{3/2}^{\tilde{a}}$ is a branching ratio of axino decay to gravitino. As one always finds $B_{3/2}^{\tilde{a}}/f_a^2 \gtrsim 10^{-24}$ GeV^{-2} for $f_a \lesssim 10^{12}$ GeV, this can give more stringent constraint on T_R together with eq.(71). The allowed region for the reheating temperature is

$$T_R < m_{\tilde{a}}, \quad \text{or} \quad m_{\tilde{a}} < T_R < 8.1\times 10^5\text{GeV}\left(\frac{f_a}{10^{12}\text{GeV}}\right)\left(\frac{600\text{GeV}}{M_{\tilde{g}}}\right)\frac{1}{\sqrt{B_{3/2}^{\tilde{a}}}}.$$

therefore the initial amplitude should be small enough. With $O(\Lambda)$ and $O(f)$ for the sizes of the initial amplitudes, the problem can be avoided.

The viable parameter regions are shown in Figures 2 and 3 which are very similar to Figure 1. Therefore, the viable range we obtain is the same as Eqs. (69) and (70). We summarize in Table 2 the numerical values of the masses and decay widths in the parameter range of our interest.

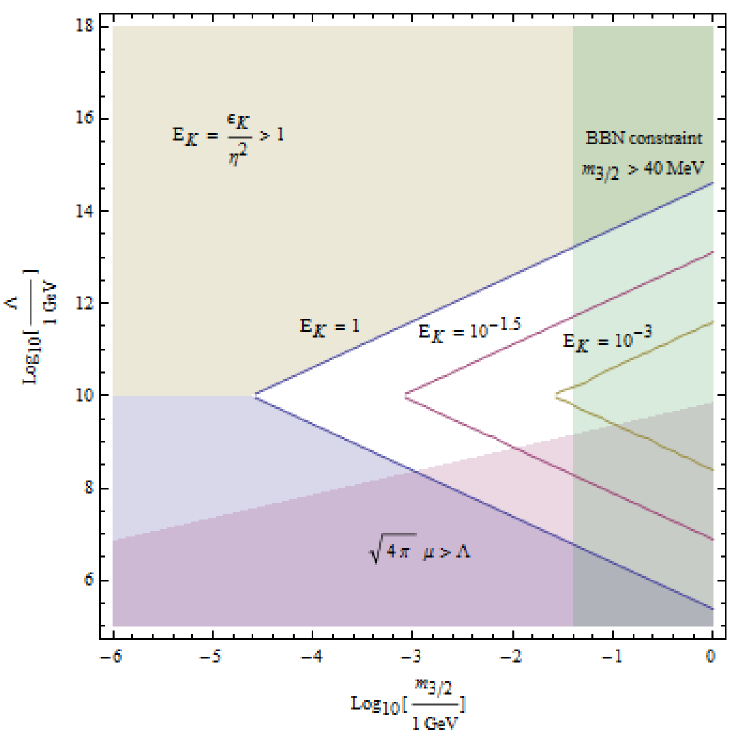

Figure 2: A figure for an allowed parameter region for Model 2. This is similar to the Model 1 except for $\epsilon_K \lesssim \eta^2$. Here we defined $\eta = \frac{\Lambda_c}{\Lambda}$ and $E_K = \epsilon_K \eta^{-2}$.

5 Brief comments on axions in IIB orientifold/F-theory GUTs

In string therory, besides the field theoretic axions, we often obtain very light string theoretic axions via moduli stabilization in the low scale SUGRA [58, 59]. In general, the number of axions is estimated as

(The number of axions) = (The number of moduli fields) $+1-$ (The number of terms in the W).

Here W is the superpotential and a factor unity comes from the R-symmetry. This is because PQ symmetries of (moduli) fields and the R-symmetry produce candidates of the axion whereas

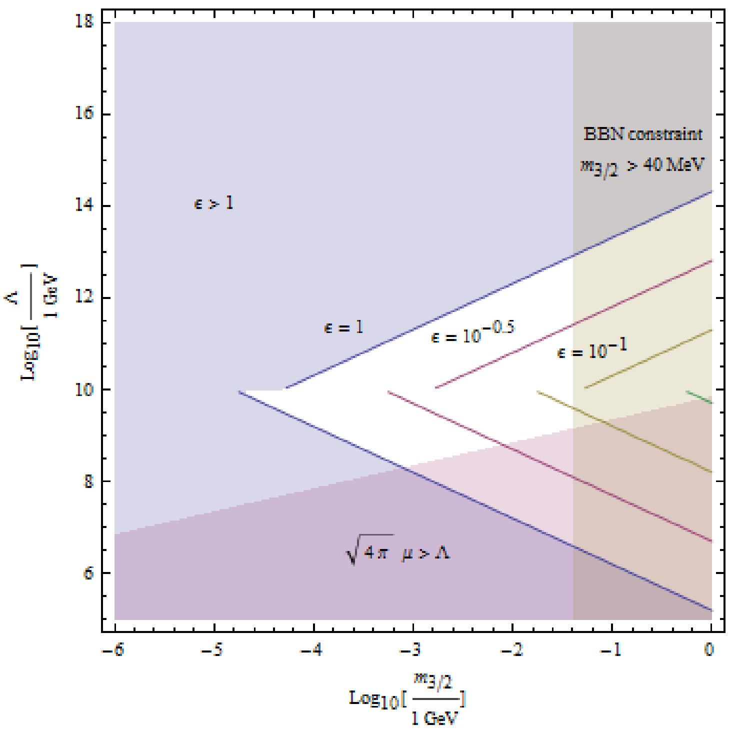

Figure 3: The similar figure for Model 3. This is similar to the Model 1 and 2 except for $\epsilon \lesssim 1$. The discontinuity of ϵ originates from the fact $F^A \propto \tilde{d} = (d_1 + 2q_X)$ and we took $q_X = 1$ for $f > \Lambda$ while $q_X = 0$ for $f < \Lambda$.

	Model 1	Model 2	Model 3
m_{X_R} (GeV)	$10^5 - 10^8$	$10^4 - 10^8$	$10^4 - 10^8$
$\Gamma_{X_R} \simeq \Gamma_{X_R \to 2\psi_{3/2}}$ $(+\Gamma^{\rm M1}_{X_R \to 2X_I})$ (GeV)	$10^{-10} - 10^4$	$10^{-16} - 10^4$	$10^{-16} - 10^4$
m_{X_I} (GeV)	$5 - 5 \times 10^3$	$10^4 - 10^8$	$10^4 - 10^8$
$\Gamma_{X_I} \simeq \Gamma^{\rm M1}_{X_I \to 2\lambda}$ or $\Gamma^{\rm M1}_{X_I \to 2b}$ or $\Gamma^{\rm M2,3}_{X_I \to 2\psi_{3/2}}$ (GeV)	$10^{-24} - 10^{-11}$	$10^{-16} - 10^4$	$10^{-16} - 10^4$
m_{A_R} (GeV)	$10^5 - 10^7$	$10^4 - 10^8$	$10^4 - 10^8$
$\Gamma_{A_R} \simeq \Gamma_{A_R \to 2a} + \Gamma_{A_R \to \psi_{3/2}\tilde{a}}$ (GeV)	$10^{-7} - 10$	$10^{-10} - 10$	$10^{-10} - 10$
$m_{\tilde{a}}$ (GeV)	$5 \times 10^4 - 10^6$	10^4	$10^4 - 10^6$
$\Gamma_{\tilde{a}} \simeq \Gamma_{\tilde{a} \to \psi_{3/2} a} + \Gamma_{\tilde{a} \to \lambda g}$ (GeV)	$10^{-12} - 10^{-5}$	$10^{-12} - 10^{-10}$	$10^{-12} - 10^{-6}$
R-breaking parameter: $(x_0, \frac{\epsilon_K}{\eta^2}, \epsilon)$	$10^{-0.8} - 1$	$10^{-3} - 1$	$10^{-1} - 1$

Table 2: A table of examples of numerical values in each model. We took all dimensionless parameters as of order unity, $M_{1/2} = 100$ GeV, $f = 10^{10}$ GeV and $m_{3/2} \lesssim 40$ MeV. Here we defined $\eta = \Lambda_c/\Lambda$.

independent terms in the superpotential kill them, supposing the Kähler potential K preserves these symmetries. When this counting gets negative or zero, we do not have any light axions. If there are very small subleading terms violating PQ symmetries in W or K, they give very light mass to the axions. If we are to identify one of the axions with the QCD axion, the quality of the PQ symmetry needs to be checked for solving the strong CP problem: $\delta m_a^2 \lesssim 10^{-11}(m_a^{\rm QCD})^2$. Here axion mass δm_a^2 is a contribution from non-QCD effects and $(m_a^{\rm QCD})^2$ is the QCD axion mass just from the instanton.

For a string theoretic (QCD) axion, we often encounter a logarithm type Kähler potential like $K_{\rm axion} = -M_{\rm Pl}^2 \log(A+A)$, while we can see also quadratic Kähler potential $K_{\rm axion} = \frac{f_s^2}{2}(A+A^\dagger)^2$ near the singularity. For logarithm case, by the expansion around the vev like the standard form, we can obtain the quadratic one with a decay constant $f = \frac{M_{\rm Pl}}{(\langle A+A^\dagger \rangle)} \sim f_s = O(M_{\rm string})$, where $M_{\rm string}$ is the string scale. Then we can also obtain $K = (X^\dagger X)^2/f^2$ after integrating out a massive gauge boson of (non-)anomalous $U(1)_{\rm PQ}$[15]. Its contribution to the X mass is on the

[15] For non-anomalous $U(1)$ the gauge boson mass is lower than the string scale bacause a matter-like field always

order of F^X/f. This corresponds to the case with $\Lambda \sim f$, once we could manage to obtain f at an intermediate scale. One should also consider the cosmological constraints since there typically exist light moduli in a low scale SUSY breaking scenario [38]. (See also a recent paper [60].)

In Ref. [35], it has been studied a model which has the same superpotential as in Eq. (4). There, $U(1)_{\rm PQ}$ is an anomalous gauge symmetry at a high energy scale, and X is the Polonyi field. A is a string theoretic axion multiplet and is describing the 4-cycle volume on which a D7-brane holding the $U(1)_{\rm PQ}$ is wrapping. In the paper $\langle X\rangle = f_a/\sqrt{2}$ is obtained and A is absorbed into the $U(1)_{\rm PQ}$ gauge multiplet via D-term moduli stabilization since we have $f_a \ll f_A$. Here f_A is a decay constant of A. Hence, at low energy, the relevant field is only the Polonyi field, X; the field $\mathrm{Arg}(X)$, the R-axion, is identified as the QCD axion. However, from the discussion in Refs. [20] and [61], the axion becomes either massive or the decay constant is unacceptably large. Therefore we need to modify the superpotential to have the QCD axion successfully.

For this purpose, we may consider an effective superpotential on intersecting seven branes induced by stringy instantons [62], for instance:

$$W \;=\; \epsilon M_{\rm Pl} s X e^{-q_B B} + X'(s\cdot s' - \epsilon' M_{\rm Pl}^2 e^{-q_B B}) + s'\Psi\bar{\Psi} + \cdots \tag{75}$$

with $U(1)_{\rm PQ}\times U(1)_{\rm PQ'}$ symmetry where both $U(1)$ symmetries are anomalous. Here Ψ and $\bar{\Psi}$ are messenger fields, dots represent constant W_0, heavy fields in the SUSY breaking sector or in the PQ breaking sector, the moduli stabilization sector and so on. X' is a Lagrange multiplier superfield. ϵ and ϵ' are complex structure moduli/dilaton and other moduli contributions[16] besides B. B is now a string theoretic axion multiplet describing a proper 4-cycle which would not intersect with a D7-brane holding $U(1)_{\rm PQ}$ (or its orientifold image brane), and transforms non-linearly under $U(1)_{\rm PQ'}$; we may find $PQ(e^{-q_B B}) = PQ(X') = 0$, $PQ(X) = q_X = -q_{\Psi\bar{\Psi}}$ and $q_B = PQ'(s) + PQ'(X) = PQ'(X') \neq 0$. s and s' may appear as $s = f_A e^{-q_X A}$ and $s' = f_A e^{q_X A}$ through a constraint where we defined $M_{\rm Pl}^2\langle \epsilon' e^{-q_B B}\rangle \equiv f_A^2 = O(f_a^2)$ and we obtain $X' = 0$ via an F-term equation. Stabilization of these fields, however, can depend on the model. For instance, let $\xi_{U(1)_{\rm PQ}}$ and $\xi_{U(1)_{\rm PQ'}}$ be Fayet-Iliopoulos (FI) terms in $U(1)_{\rm PQ}$ and $U(1)_{\rm PQ'}$ respectively. Note $\xi_{U(1)_{\rm PQ'}}$ includes B while $\xi_{U(1)_{\rm PQ}}$ can depend on other modulus, say C. Then D-term potential (with non-trivial world volume flux) would almost fix C and B at $\xi_{U(1)_{\rm PQ}} \simeq 0$

cancels its Fayet-Iliopoulos term; in general we obtain $K = (X^\dagger X)^2/m_{U(1)}^2$, where $m_{U(1)}$ is the (non-)anomalous $U(1)$ gauge boson mass.

[16] We will assume ϵ and ϵ' are singlet under two $U(1)$s for simplicity below, and they might become important to get proper scales. On the other hand, instead of $\epsilon' e^{-q_B B}$, we could have an another moduli contribution $\epsilon'' e^{-q_{B'} B'}$ ($B' \neq B$) for instance.

and $\xi_{U(1)_{\rm PQ'}} \simeq 0$, leaving X and A light modes[17]. Thus each $U(1)$ gauge multiplet would get massive by eating B and C, assuming decay constants of B and C are of $O(M_{\rm string})$ which are larger than f_A and $|\langle X\rangle|$. Hence we would have the global $U(1)_{\rm PQ}$ symetries in the low energy scale[18], though another global $U(1)_{\rm PQ'}$ will be broken by a stringy instanton. Then we could gain $W = W_0 + \mu^2 X e^{-q_X A}$ in the low energy. Further analyses would be beyond our scope since they depend on string model including issues of string moduli stabilization (see for instance [21]).

Finally, we would like to mention moduli stabilization in type IIB orientifold models. For moduli mediation scenario, we can get (more than) TeV scale moduli masses. This is because moduli masses are typically related with SUSY breaking gravitino mass because of axionic shift symmetries. They should be heavy enough to avoid cosmological problems. For gauge mediation scenario, we may need light gravitino mass and may get subsequently light moduli. Hence, for such a case with a field theoretic QCD axion, supersymmetric moduli stabilization which breaks the shift symmetry should be considered, such that we obtain heavy moduli: $\langle W\rangle \approx 0$, $\langle \partial_{\rm moduli} W\rangle \approx 0$ via fine-tuning [63]. For instance, $W = W_0 + \sum_i^{h^+_{1,1}} (A_i e^{-a_i T_i} + B_i e^{-b_i T_i})$ with a fine-tuning of fluxes $W_0 < 1$ or non-geometric flux compactifications [64] might be viable. To obtain large volume, $T_i \gg 1$, we will need further fine-tunings among A_i, B_i, a_i, and b_i. The remaining Kähler moduli may be stabilized via D-terms as mentioned above. Then we get $F^{\rm moduli} = 0$ and much smaller constant term in the superpotential to obtain light gravitino mass, and moduli mass are decoupled to the gravitino mass: $\langle W\rangle \ll \langle \partial^2_{\rm moduli} W\rangle$ in the Planck unit.

6 Discussion and conclusion

We discussed supersymmetric effective theories of the axion field and the Polonyi field, which can be charged under global $U(1)_{\rm PQ}$ symmetry. We assumed that the SUSY breaking and the PQ breaking sectors are directly coupled whereas the visible sector is communicate with those sectors through messenger fields. For a concreteness, we construct theories by Taylor expansion in X and A. To compute those in a simpler way or as generalizations, we transformed theories to "the standard form" with (partial) $U(1)_{\rm PQ}$ transformation after expansion around the saxion

[17] We assumed an instanton depending on C does not have two fermionic zero modes related with $\mathcal{N} = 1$ supercharge broken by the instanton or its coefficient in the superpotential has a vanishing vev; $\partial_C K = 0$ could mean the potential minimum and $\partial_B K W + M_{\rm Pl}^2 \partial_B W = 0$ would be simultaneously satisfied there, though $\xi \lesssim f_A^2$ would be sufficient for us to get a proper axion decay constant.

[18] We could have another possibilities without using an anomalous $U(1)$ gauge symmetry. For accidental axion models in heterotic orbifold, see [18]. For a discrete R-symmetry argument which heterotic string models might have, see [23].

vev $\langle A_0 \rangle$. Then we considered three R-breaking models to obtain large gaugino mass in a gauge mediation with light gravitino mass such like $m_{3/2} \lesssim 40$ MeV to avoid BBN constraints. In those models, axion multiplet plays important roles in mediating axion and SUSY breaking to the visible sector. Both the Polonyi and saxion can be heavy because the former has the low scale cut-off Λ and the latter has a direct coupling to the Polonyi field. However, they must not dominate the universe after the inflation. If the initial amplitude is of the order of the cut-off scale of the effective theory, such dominations can be avoided. For $T_R \lesssim O(10^6)$ GeV or $T_R \lesssim m_{\tilde{a}}$, the axion or gravitino can be dark matter of the universe.

We have several things we did not consider here explicitly. For instance, since inflaton can decay to gravitinos [65], the resulting gravitino abundance may give an affect on our study. But this can depend on the inflation model. We will also need to discuss the generation of μ-term and $B\mu$-term, which is related to the main decay mode of the R-axion and other fields. This issue is common in gauge mediation models. (See recent gauge mediation models with the QCD axion [22], in which messenger fields of SUSY breaking and axion are not unified. With $W_\mu = \mu' e^{-qA} H_u H_d$ [5] and with $F^A \simeq 0$ up to $m_{3/2}$, $\mu/B\mu$ problem have been solved there.)

As future directions, we can also study another parameter region, where some dimensionless parameters are much smaller than of order unity. For such a case, new possibilities may open up; for instance, we may have much smaller masses of saxion and axino than our cases. The saxion dominated universe may be allowed in such cases because of the different decay properties. It will be also interesting to study the UV completion, where the SUSY breaking sector and the PQ breaking sector are unified. That will be a minimal model to solve hierarchy problem and strong CP problem.

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Appendix

A Supergravity effects

In SUGRA, R-symmetry can be broken by a constant term in the superpotential, $\delta W = \mu^2 W_0$. The scalar potential is

$$V_{\rm SUGRA} = e^{K/M_{\rm Pl}^2}\left[K^{i\bar{j}} D_i W \overline{D_j W} - 3\frac{|W|^2}{M_{\rm Pl}^2}\right], \tag{76}$$

where $D_i W = \partial_i W + W\frac{(\partial_i K)}{M_{\rm Pl}^2}$. The gravitino mass is given by $m_{3/2} = \mu^2 W_0/M_{\rm Pl}^2$. Here the Kähler potential and the superpotential are given by eq.(16) and (17). Hereafter in the appendix we will define

$$\eta \equiv \frac{\Lambda_c}{\Lambda}. \tag{77}$$

- R-symmetric case: $a = 1$

  In this case, the vev shift is

  $$\delta X = \frac{2\Lambda}{\sqrt{3}M_{\rm Pl}} \cdot \frac{\sqrt{3}W_0}{M_{\rm Pl}}, \quad \delta A = -\frac{c_0\left(\eta^{-2} f^2 + 2\tilde{c}\Lambda^2\right)}{3\tilde{c}M_{\rm Pl}^2}.$$

  Here c_0 is a coefficient of the linear term in the Kähler potential, $c_0 f^2(A + A^\dagger)$, and in order to make our perturbation valid, we need $\frac{\Lambda}{M_{\rm Pl}} \ll \frac{f}{\Lambda}$ for $f \ll \Lambda$. The constant term W_0 is fixed by the condition $V_{\rm SUGRA} = 0$:

  $$W_0 \simeq \frac{1}{\sqrt{3}} M_{\rm Pl}. \tag{78}$$

  Then we find $\Lambda F_X = -\mu^2 = -\sqrt{3} m_{3/2} M_{\rm Pl}$. The axino obtains a mass

  $$m_{\tilde{a}} = m_{3/2}\left(1 + \eta^2 \frac{2\tilde{c}\Lambda^2}{f^2}\right). \tag{79}$$

  The first $m_{3/2}$ contribution vanishes when we replace the axion kinetic term, $(A+A^\dagger)^2/2$, with $\exp(A+A^\dagger)$. This is because we can define $A_{\rm new} = A + \log\Phi_c$, where Φ_c is a conformal compensator. Hence loop effects, such that $c_1'(A + A^\dagger)\cdot\exp(A + A^\dagger)$, are important. The axion field A gets an F-component vev at the leading order of $1/M_{\rm Pl}$:

  $$F^A = c_0 m_{3/2}. \tag{80}$$

- Spontaneous R-symmetry breaking case: $a = -1$ and $x_0 \neq 0$

  There are similar vev shifts and F^A to the previous case:

  $$\delta X = \frac{\Lambda}{\sqrt{3}M_{\rm Pl}} + \frac{c_0 q_X}{3\tilde{c}}\left(\frac{f}{\eta M_{\rm Pl}}\right)^2 x_0, \quad \delta A = -\frac{c_0(f^2\eta^{-2} + \tilde{c}\Lambda^2)}{3\tilde{c}M_{\rm Pl}^2} - \frac{(q_X\eta^{-2} + c_0\tilde{c})}{\sqrt{3}\tilde{c}}\frac{x_0\Lambda}{M_{\rm Pl}},$$

$$F_A = c_0 m_{3/2}. \tag{81}$$

Here we used $W_0 = M_{\rm Pl}/\sqrt{3}$. Recall that we will have $F_A \propto x_0^3$ term, which is not suppressed by $M_{\rm Pl}$. The R-axion gets a mass:

$$m_{X_I}^2 = \frac{2\mu^4}{\sqrt{3}x_0 \Lambda M_{\rm Pl}} = 2\sqrt{3} m_{3/2}^2 \left(\frac{M_{\rm Pl}}{x_0 \Lambda}\right) \tag{82}$$

at the leading order in x_0 and Planck suppressed expansions. Now the axino mass is given by

$$m_{\tilde{a}} = m_{3/2} + \frac{\eta^2 \tilde{c} \mu^2 x_0 \Lambda}{f^2} = m_{3/2}\left(1 + \sqrt{3}\eta^2 \tilde{c} \frac{x_0 \Lambda M_{\rm Pl}}{f^2}\right). \tag{83}$$

The first $m_{3/2}$ contribution vanishes again when we replace the axion kinetic term, $(A + A^\dagger)^2/2$, with $\exp(A + A^\dagger)$.

B Solving mixing between X and A

B.1 Kinetic mixing in the standard form by the superfield description: axion mixing and fermion mixing

Let us consider the following Kähler potential, which preserves $U(1)_{\rm PQ}$ symmetry

$$K = X^\dagger X + \frac{f^2}{2}(A + A^\dagger)^2 + K_{XA^\dagger} X (A + A^\dagger) + K_{AX^\dagger}(A + A^\dagger) X^\dagger. \tag{84}$$

As we want to solve kinetic mixing, we will focus on the vacuum in which $K_{XA^\dagger}$ has the vev:

$$K \to K_0 = X^\dagger X + \frac{f^2}{2}(A + A^\dagger)^2 + (A + A^\dagger)(\kappa X + \kappa^* X^\dagger). \tag{85}$$

Here we defined $\kappa \equiv \langle K_{XA^\dagger}\rangle$. After solving mixing between A and X, the above Kähler potential becomes

$$K_0 = \left(1 - \frac{|\kappa|^2}{f^2}\right) X^\dagger X + \frac{f^2}{2}(\hat{A} + \hat{A}^\dagger)^2 + (\text{holomorphic term} + c.c.). \tag{86}$$

A diagonalized axion $\hat{A}$ is given by

$$\hat{A} = A + \frac{\kappa X}{f^2} = A + \frac{\langle K_{XA^\dagger}\rangle X}{f^2}. \tag{87}$$

Thus when one considers the canonical normalization we find

$$k \cdot A \to \frac{\sqrt{2}\hat{A}}{f_a} - \frac{\sqrt{2}X}{f_y}. \tag{88}$$

Here

$$f_a = \sqrt{2}\frac{f}{k}, \quad f_y = \frac{1}{k}\sqrt{2}f\left(\frac{f}{\langle K_{XA^\dagger}\rangle}\right)\sqrt{1-\frac{|\langle K_{XA^\dagger}\rangle|^2}{f^2}}. \tag{89}$$

With a gauge kinetic term $S = \frac{1}{2g^2} + k\frac{A}{8\pi^2}$, we obtain

$$S = \frac{1}{2g^2} + \frac{1}{8\pi^2}\left(\frac{\sqrt{2}\hat{A}}{f_a} - \frac{\sqrt{2}X}{f_y}\right). \tag{90}$$

Furthermore, in the standard form, F_X and F-component of A is given by

$$F_X \simeq -(\partial_X W)^\dagger, \tag{91}$$
$$F_A = -K^{AX^\dagger}(\partial_X W)^\dagger \simeq -\frac{K_{XA^\dagger}}{f^2}F_X. \tag{92}$$

Here we used $K_{XX^\dagger} \simeq 1,\ K_{AA^\dagger} \simeq f^2$ and $K_{XX^\dagger}K_{AA^\dagger} \gg K^2_{XA^\dagger}$ for $f \gg K_{XA^\dagger}$.

Thus, with regard to f_y, we can get

$$f_y \simeq \frac{1}{k}\sqrt{2}\left(\frac{f^2}{\langle K_{XA^\dagger}\rangle}\right) \simeq -\frac{1}{k}\sqrt{2}\frac{\langle F_X\rangle}{\langle F_A\rangle} \simeq \frac{\alpha}{4\pi}\frac{\sqrt{6}m_{3/2}M_{\rm Pl}}{M_{1/2}}. \tag{93}$$

Here we used $\langle F_X\rangle \simeq -\sqrt{3}m_{3/2}M_{\rm Pl}$, $\langle F_A\rangle \simeq \frac{4\pi}{k\alpha}M_{1/2}$. Thus we can find

$$\frac{f_a}{f_y} = -\frac{fF^A}{F^X} \equiv -\tan\Theta, \tag{94}$$

where Θ is a goldstino angle.

Using this goldstino angle, we obtain goldstino ψ_X and axino $\tilde{a}$

$$\psi_X \simeq \psi_X^{(0)} - \frac{f_a}{f_y}\psi_A^{(0)} \quad \tilde{a} \simeq \psi_A^{(0)} + \frac{f_a}{f_y}\psi_X^{(0)}, \tag{95}$$

where we assumed $\Theta \ll 1$. In the above equation, we normalized fermions canonically and $\psi^{(0)}$ means the original field before solving the mixing. Thus we find $\langle \delta_{\rm SUSY}\tilde{a}\rangle \sim \langle F_{\hat{A}}\rangle = 0$.

B.2 Mixing between X_R and A_R

Remaining fields to be solved are X_R and A_R. Now there is also mass mixing in the scalar potential in addition to the kinetic mixing:

$$V = \frac{1}{2}m_{X_R}^2 X_R^2 + \frac{1}{2}m_{A_R}^2 A_R^2 + m_{XA}^2 X_R A_R, \tag{96}$$

Here recall that we already solved the kinetic mixing with original field $(X_R^{(0)},\ A_R^{(0)})$: $X_R = X_R^{(0)},\ A_R = A_R^{(0)} + \frac{f_a}{f_y} X_R^{(0)}$. By an unitary rotation, we will obtain diagonalized fields $(\hat{X}_R,\ \hat{A}_R)$ with canonical and diagonal kinetic term:

$$V \approx \frac{1}{2} m_{X_R}^2 (\hat{X}_R)^2 + \frac{1}{2} m_{A_R}^2 (\hat{A}_R)^2. \tag{97}$$

Here

$$\begin{aligned}
\begin{pmatrix} X_R^{(0)} \\ \frac{A_R^{(0)}}{f_a} \end{pmatrix} &= \begin{pmatrix} 1 & -\frac{m_{XA}^2}{M^2} f_a \\ \frac{m_{XA}^2}{M^2 f_a} - \frac{1}{f_y} & 1 \end{pmatrix} \begin{pmatrix} \hat{X}_R \\ \frac{\hat{A}_R}{f_a} \end{pmatrix}, \quad M^2 = |m_{X_R}^2 - m_{A_R}^2| \ \text{ for } m_{X_R} \neq m_{A_R}, \\
&= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -f_a \\ \frac{1}{f_a} - \frac{1}{f_y} & 1 + \frac{f_a}{f_y} \end{pmatrix} \begin{pmatrix} \hat{X}_R \\ \frac{\hat{A}_R}{f_a} \end{pmatrix} \quad \text{for } |m_{X_R}^2 - m_{A_R}^2| < m_{XA}^2 \qquad (98) \\
&\equiv \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix} \begin{pmatrix} \hat{X}_R \\ \frac{\hat{A}_R}{f_a} \end{pmatrix}. \qquad (99)
\end{aligned}$$

Here we will always have $m_{X_R} \gtrsim m_{A_R}$ because of Λ_c/Λ; hereafter we will not consider the degenerate case.

For Model 0, we will find

$$\frac{m_{XA}^2}{m_{X_R}^2} \sim \frac{\Lambda_c^2}{f M_{\rm Pl}} \sim \left(\frac{\Lambda_c}{f}\right)^2 \frac{f_a}{f_y} \leq \frac{f_a}{f_y}. \tag{100}$$

Here $f_y \sim M_{\rm Pl}$.

B.2.1 Mixing between X_R and A_R in Model 1

In this case, we have

$$\begin{aligned}
m_{X_R}^2 &= 2\frac{\mu^4}{\Lambda^2}, \quad m_{A_R}^2 = 2\eta^2 \tilde{c} \frac{\mu^4}{f^2}, \qquad (101) \\
m_{XA}^2 &= \eta^2 \frac{2(-\tilde{d} + \tilde{e})\mu^4 x_0}{f\Lambda} - \frac{f_a}{f_y} m_{A_R}^2 \qquad (102) \\
&\sim k m_{X_R}^2 \frac{f_a m_{\tilde{a}}}{\sqrt{6} m_{3/2} M_{\rm Pl}} \left(1 + \frac{\frac{4\pi}{k\alpha} M_{1/2}}{m_{\tilde{a}}} \frac{m_{A_R}^2}{m_{X_R}^2}\right) \qquad (103) \\
&\simeq 2.8 \times 10^{12} \text{GeV}^2 \left(\frac{m_{X_R}}{10^7 \text{GeV}}\right)^2 \left(\frac{m_{\tilde{a}}}{5 \times 10^5 \text{GeV}}\right) \left(\frac{f_a}{10^{10} \text{GeV}}\right) \left(\frac{30 \text{MeV}}{m_{3/2}}\right).
\end{aligned}$$

Here we supposed $(-\tilde{d} + \tilde{e}) \sim \tilde{c}$. Note that mass mixing in the scalar potential m_{XA}^2 is of $O(x_0)$ while kinetic mixing f_y^{-1} is of $O(x_0^3)$.

Thus

$$\frac{m_{XA}^2}{m_{X_R}^2} \simeq k \frac{f_a m_{\tilde{a}}}{\sqrt{6} m_{3/2} M_{\rm Pl}} \simeq 2.8 \times 10^{-2}. \tag{104}$$

Here we used used $m_{X_R} \gtrsim m_{A_R}$ and took $f_a = 10^{10}$ GeV, $m_{\tilde{a}} = 5 \times 10^5$ GeV and $m_{3/2} = 30$ MeV.

B.2.2 Mixing between X_R and A_R in Model 2

Recall that eq.(41):

$$
\begin{aligned}
m_{XA}^2 &= \epsilon_K \frac{\Lambda}{2f}\frac{\mu^4}{f^2}\left(-6\eta^2\tilde{c} + \frac{f^2}{\Lambda^2}\right) && (105)\\
&\equiv \frac{4\pi}{\alpha}\frac{M_{1/2}f_a}{\sqrt{6}m_{3/2}M_{\rm Pl}}Cm_X^2 && (106)\\
&\simeq k\frac{m_{\tilde{a}}f_a}{\sqrt{6}m_{3/2}M_{\rm Pl}}Cm_X^2. && (107)
\end{aligned}
$$

Here $C = O(1)$, $m_X^2 = \mu^4/\Lambda^2$, and we used $F^A \simeq m_{\tilde{a}} \simeq \frac{4\pi}{k\alpha}M_{1/2}$. Then the magnitude of the mass mixing is the same order as that of the kinetic mixing:

$$
\begin{aligned}
\frac{m_{XA}^2}{m_X^2} &\simeq k\frac{m_{\tilde{a}}f_a}{\sqrt{6}m_{3/2}M_{\rm Pl}}C && (108)\\
&\simeq C\frac{f_a}{f_y} \sim 10^{-3} \times C. && (109)
\end{aligned}
$$

Here we took $m_{3/2} = 30$ MeV and $m_{\tilde{a}} \sim 10^4$ GeV.

B.2.3 Mixing between X_R and A_R in Model 3

In this case, we have the similar situation to the Model 1, that is, the effect of the mass mixing is larger than that of the kinetic mixing:

$$
\begin{aligned}
m_{XA}^2 &\simeq -2\epsilon\eta^2\frac{\tilde{d}}{\Lambda f}\mu^4 && (110)\\
&\sim -2k\frac{m_{\tilde{a}}}{F^A}\frac{f_a}{f_y}m_X^2 \quad \text{for} \quad \tilde{d} \sim \tilde{c} && (111)
\end{aligned}
$$

Then

$$
\begin{aligned}
\frac{m_{XA}^2}{m_X^2} &\sim -2k\frac{m_{\tilde{a}}f_a}{\sqrt{6}m_{3/2}M_{\rm Pl}} && (112)\\
&\simeq -5.3 \times 10^{-2}. && (113)
\end{aligned}
$$

Here we took $m_{\tilde{a}} = 10^5$ GeV and $m_{3/2} = 30$ MeV.

C Derivatives of F^X and F^A

We need computations of $\partial_I F^J$ $(I, J = X,\ X^\dagger,\ A,\ A^\dagger)$ to obtain interactions between gaugino and those fields for instance. Now we will focus on the gauge kinetic term:

$$\frac{1}{2}\int d\theta^2 S W^\alpha W_\alpha + c.c., \quad S = \frac{1}{2g^2} + k\frac{A}{8\pi^2}. \tag{114}$$

As we have a gaugino mass from F^A, we can read a coupling of X to gaugino pair from the gaugino mass at the leading order of fluctuation $\delta_f X$:

$$\begin{aligned}
\frac{1}{2}M_{1/2} &= \frac{1}{2}F_A\partial_A \log(\mathrm{Re}(S)(A)) && (115)\\
&\simeq k\frac{\alpha}{4\pi}\frac{K_{XA^\dagger}}{f^2}(\partial_X W)^\dagger && (116)\\
&= \frac{1}{2}\langle M_{1/2}\rangle\left(1 + \delta_f X\langle\partial_X \log\left(K_{XA^\dagger}\right)\rangle\right) + \cdots && (117)\\
&\sim \frac{1}{2}\langle M_{1/2}\rangle\left(1 + \frac{\delta_f X}{\langle X\rangle}\right) + \ldots. && (118)
\end{aligned}$$

Here we assumed $K_{XA^\dagger}$ is a polynomial in X, so we can gain $\langle\partial_X \log\left(K_{XA^\dagger}\right)\rangle \sim 1/\langle X\rangle$ unless we have a small X vev or cancellations like Model 2 below. We used a notation $\langle M_{1/2}\rangle$ to distinguish dynamical fields from a parameter.

C.1 Derivatives of F^A in Model 0

$$\begin{aligned}
\frac{1}{3}\partial_X F^X &\simeq \frac{1}{2}\partial_{X^\dagger}F^X \simeq -m_{3/2}, \quad \partial_A F^X = \partial_{A^\dagger}F^X \sim \frac{f^2}{\Lambda}\frac{m_{3/2}}{M_{\mathrm{Pl}}}, && (119)\\
\partial_X F^A &\sim \frac{m_{3/2}}{M_{\mathrm{Pl}}}\left(\frac{m_{A_R}}{m_X}\right)^2\Lambda, \quad \partial_{X^\dagger}F^A \sim \frac{m_{3/2}}{M_{\mathrm{Pl}}}\Lambda, \quad \partial_A F^A = \partial_{A^\dagger}F^A \sim m_{3/2}. && (120)
\end{aligned}$$

C.2 Derivatives of F^X and F^A in Model 1

$$\begin{aligned}
\partial_X F^X &= \partial_{X^\dagger}F^X \sim F^A x_0^2\left(\eta^2 + \left(\frac{f}{\Lambda}\right)^2\right), \quad \partial_A F^X = \partial_{A^\dagger}F^X \sim F^A x_0\left(\frac{f}{\Lambda}\right)^2. && (121)\\
\partial_X F^A &= -\frac{F^A}{x_0}, \quad \partial_{X^\dagger}F^A \sim F^A x_0, \quad \partial_A F^A = \partial_{A^\dagger}F^A = m_{\tilde{a}} && (122)
\end{aligned}$$

C.3 Derivatives of F^X and F^A in Model 2

$$\begin{aligned}
\partial_X F^X &= \partial_{X^\dagger} F^X \sim F^A \left(\frac{\epsilon_K}{\eta^2}\right)^4 \eta^6 \left(\frac{f}{\Lambda}\right)^4, \quad \partial_A F^X = \partial_{A^\dagger} F^X \sim F^A \left(\frac{\epsilon_K}{\eta^2}\right)^5 \eta^6 \left(\frac{f}{\Lambda}\right)^4 &(123)\\
\partial_X F^A &\sim F^A \left(\frac{\epsilon_K}{\eta^2}\right)^3 \eta^8 \left(\frac{f}{\Lambda}\right)^6, \quad \partial_{X^\dagger} F^A = -\epsilon_K^2 C_3 \frac{\Lambda^3 \mu^2}{f^4} \simeq -\sqrt{2}\Lambda \frac{m_{\tilde{a}}}{f_y}\frac{1}{k} &(124)\\
\partial_A F^A &= \partial_{A^\dagger} F^A = -C_3 F^A \simeq -m_{\tilde{a}}. &(125)
\end{aligned}$$

C.4 Derivatives of F^X and F^A in Model 3

$$\begin{aligned}
\partial_X F^X &= \frac{\epsilon\mu^2}{\Lambda} \sim F^A \left(\frac{f}{\epsilon\Lambda}\right)^2, \quad \partial_{X^\dagger} F^X = -\frac{\epsilon^3\mu^2}{\Lambda} \sim F^A \left(\frac{f}{\Lambda}\right)^2 &(126)\\
\partial_A F^X &= \partial_{A^\dagger} F^X = k^2 \frac{f_a^2}{\delta X \Lambda^2} F^A &(127)\\
\partial_X F^A &= \frac{1}{2}\partial_{X^\dagger} F^A = \frac{F^A}{\delta X}, \quad \partial_A F^A = \partial_{A^\dagger} F^A = m_{\tilde{a}}. &(128)
\end{aligned}$$

D Some decay modes

Here we will exhibit several decay modes. The results can be rough and just show their order of magnitude.

D.1 The Polonyi and the R-axion decay

- $\Gamma(X_R \to 2X_I)$ in Model 1

  Through an interaction $\frac{X_R}{\sqrt{2}x_0\Lambda}\partial_\mu X_I \partial^\mu X_I$ in the Kähler potential $K = \Lambda^2 X^\dagger X$, we can obtain

$$\Gamma(X_R \to X_I X_I) \simeq \frac{1}{64\pi}\frac{m_{X_R}^3}{(x_0\Lambda)^2}. \tag{129}$$

- $\Gamma(X \to 2\psi_{3/2})$

  With an interaction $K = \Lambda^2 \frac{(X^\dagger X)^2}{4}$ or $K = \Lambda^2 X^\dagger X$ for the R-axion in Model 1[19], we can find

$$\Gamma(X \to \psi_{3/2}\psi_{3/2}) \simeq \frac{1}{96\pi}\frac{m_X^5}{m_{3/2}^2 M_{\rm Pl}^2}. \tag{130}$$

[19]We have an interaction between goldstino ψ_X and R-axion X_I through $K = \Lambda^2 X^\dagger X$: $\frac{1}{\sqrt{2}x_0\Lambda}\partial_\mu X_I \bar{\psi}_X \bar{\sigma}^\mu \psi_X$. A contribution from $K\Lambda^{-2} = \frac{(X^\dagger X)^2}{4} - \frac{(X^\dagger X)^3}{18x_0^2}$ will vanish because of the vev $\langle X\rangle \simeq x_0$.

- $\Gamma(X \to \psi_{3/2} + \tilde{a})$

  Through an interaction K_{AX}, we obtain

$$\Gamma(X \to \psi_{3/2} + \tilde{a}) \simeq \left(\frac{U_{12}}{f_a}\right)^2 \frac{1}{96\pi} \frac{m_X^5}{m_{3/2}^2 M_{\rm Pl}^2}. \tag{131}$$

- $\Gamma(X \to 2g)$

  Through an interaction with mixing between A and X,

$$\frac{\alpha}{8\pi}\left(\frac{U_{12}}{f_a}\frac{X_R}{f_a}F_{\mu\nu}F^{\mu\nu} + \frac{f_a}{f_y}\frac{X_I}{f_a}F_{\mu\nu}\tilde{F}^{\mu\nu}\right), \tag{132}$$

  we obtain

$$\Gamma(X_R \to gg) \simeq \frac{N_g}{16\pi}\left(\frac{\alpha}{4\pi}\right)^2\left(\frac{U_{12}}{f_a}\right)^2 \frac{m_{X_R}^3}{f_a^2} \tag{133}$$

$$\Gamma(X_I \to gg) \simeq \frac{N_g}{16\pi}\left(\frac{\alpha}{4\pi}\right)^2\left(\frac{f_a}{f_y}\right)^2 \frac{m_{X_I}^3}{f_a^2}. \tag{134}$$

  For Model 0, we will not have a loop factor.

- $\Gamma(X \to 2\tilde{a})$

  Through an interaction

$$m_{\tilde{a}}\frac{X}{\sqrt{2}\langle X\rangle}\tilde{a}\tilde{a} \quad \text{for Model } 1, 3, \tag{135}$$

$$m_{\tilde{a}}\frac{X}{kf_y}\tilde{a}\tilde{a}\left(1 + \frac{C_3}{2}\left(\frac{m_{A_R}}{m_{\tilde{a}}}\right)^2\right) \text{for Model } 2, 0 \tag{136}$$

  we obtain

$$\Gamma(X \to \tilde{a}\tilde{a}) \simeq \frac{1}{32\pi}m_X\left(\frac{m_{\tilde{a}}}{\langle X\rangle}\right)^2 \quad \text{for Model } 1, 3, \tag{137}$$

$$\simeq \frac{1}{16\pi}m_X\left(\frac{m_{\tilde{a}}}{kf_y}\right)^2\left(1 + \frac{C_3}{2}\left(\frac{m_{A_R}}{m_{\tilde{a}}}\right)^2\right)^2 \quad \text{for Model } 2, 0. \tag{138}$$

  Note that $\langle X\rangle$ is a dimensionful vev of X and the above interactions are obtained via $\eta^2\Lambda^2\frac{\tilde{c}}{2}(A+A^\dagger)^2X^\dagger X$ and $\frac{f^2C_3}{3!}(A+A^\dagger)^3$.

- $\Gamma(X \to 2\lambda)$

  For a mode $X \to \lambda\lambda$, $\partial_X F^A$ or $\partial_{X^\dagger}F^A$ becomes a coupling of X to gaugino pair:

$$\Gamma(X \to \lambda\lambda) \simeq \frac{N_g}{32\pi}m_X\left(\frac{M_{1/2}}{\langle X\rangle}\right)^2 \quad \text{for Model } 1, 3, \tag{139}$$

$$\simeq \frac{C_3^2}{16\pi k^2}m_X\left(\frac{M_{1/2}}{f_y}\right)^2 \quad \text{for Model } 2, 0. \tag{140}$$

Thus a partial decay width of this mode can be suppressed by $\left(\frac{M_{1/2}}{m_{\tilde{a}}}\right)^2$ for Model 1 and 3 or $\left(\frac{M_{1/2}}{m_{\tilde{a}}}\right)^2\left(1+\frac{C_3}{2}\left(\frac{m_{A_R}}{m_{\tilde{a}}}\right)^2\right)^{-2}$ for Model 2 and 0, comparing with $\Gamma(X\to\tilde{a}\tilde{a})$. If we have $K=c_\mu(A+A^\dagger)H_uH_d+c_B(A+A^\dagger)^2H_uH_d$, $\Gamma(X\to\tilde{h}_u\tilde{h}_d)$ would be comparable to this mode, supposing $c_\mu\sim10^{-2}$, $c_B\lesssim10^{-4}$ and $\mu\sim M_{1/2}$.

- Γ_{X_I} in Model 1

For Model 1, we find through an interaction $-i\frac{X_I}{\sqrt{2}x_0\Lambda}\frac{M_{1/2}}{2}\lambda\lambda+c.c.$

$$\Gamma(X_I\to\lambda\lambda) \;=\; \frac{N_g}{32\pi}m_{X_I}\left(\frac{M_{1/2}}{\Lambda x_0}\right)^2\sqrt{1-\frac{4M_{1/2}^2}{m_{X_I}^2}}. \tag{141}$$

For $m_{X_I}<2M_{1/2}$, the channel $X_I\to\lambda\lambda$ is closed, then $X_I\to b\bar{b}$ can become the main decay mode:

$$\Gamma(X_I\to b\bar{b}) \;\simeq\; \frac{3m_{X_I}}{16\pi}\left(\frac{m_A^2\sin^2\beta}{x_0\Lambda}\frac{m_b}{m_A^2-m_{X_I}^2}\right)^2\sqrt{1-\frac{4m_b^2}{m_{X_I}^2}}. \tag{142}$$

For a mode of $X_I\to\tau\bar{\tau}$, m_b should be replaced with m_τ.

- $\Gamma(X_R\to2\Phi)$

We denoted MSSM scalar fields as Φ. For decay mode $X_R\to2\Phi$, $K=k\left(\frac{\alpha}{4\pi}\right)^2(A+A^\dagger)^2\Phi^\dagger\Phi$ is important. These decay occur via soft scalar mass $\frac{m_0^2}{\langle X\rangle}X_R\Phi^\dagger\Phi$ or $\frac{m_0^2}{f_y}X_R\Phi^\dagger\Phi$. These amplitude is given by

$$\Gamma(X_R\to2\Phi)\sim\frac{N_m}{N_g}\left(\frac{m_0}{M_{1/2}}\right)^2\left(\frac{m_0}{m_{X_R}}\right)^2\times\Gamma(X\to\lambda\lambda). \tag{143}$$

Here N_m is the number of these decay channels. Supposing we have $K=c_\mu(A+A^\dagger)H_uH_d+c_B(A+A^\dagger)^2H_uH_d$ and $B\mu\sim m_0^2$, $\Gamma(X\to H_uH_d)$ from this interaction would be comparable to this mode. The decay may also occur via derivative interaction $k\left(\frac{\alpha}{4\pi}\right)^2\langle A_0\rangle\left(\frac{U_{12}}{f_a}\right)\frac{X_R}{f_a}\Phi^\dagger\partial^2\Phi$ [66]. (Note that A includes $\langle A_0\rangle$.) However the effect of this interaction is much smaller than the above result. For Model 0, we will have similar result to the Model 2.

- $\Gamma(X_R\to2a)$ etc.

Through an interaction

$$\frac{U_{12}}{f_a}\frac{X_R}{kf_a}\partial_\mu a\partial^\mu a \;\simeq\; \frac{m_{\tilde{a}}}{\sqrt{6}m_{3/2}M_{\rm Pl}}X_R\partial_\mu a\partial^\mu a, \tag{144}$$

we will obtain

$$\Gamma(X_R \to aa) \ \simeq \ \frac{1}{192\pi}\frac{m_{X_R}^3 m_{\tilde{a}}^2}{m_{3/2}^2 M_{\rm Pl}^2}. \tag{145}$$

Note that the above interactions are obtained via $\eta^2\Lambda^2\frac{\tilde{c}}{2}(A+A^\dagger)^2X^\dagger X$ and $\frac{f^2C_3}{3!}(A+A^\dagger)^3$ for Model 1, 3 or via $\frac{f^2C_3}{3!}(A+A^\dagger)^3$ for Model 2. For $m_X \gg m_{A_R}$, we have the same partial decay width of $X_R \to A_R A_R$ as this.

D.2 Saxion decay

- $\Gamma(A_R \to 2a)$

  Through an interaction $\hat{C}_3\frac{A_R}{f_a}\partial_\mu a\partial^\mu a$ in the Kähler potential $\frac{C_3 f^2}{3!}(A+A^\dagger)^3$, we can compute

$$\Gamma(A_R \to a+a) \ = \ \frac{1}{32\pi k^2}C_3^2\frac{m_{A_R}^3}{f_a^2}. \tag{146}$$

- $\Gamma(A_R \to a + X_I)$

  From the previous interaction, we have

$$\Gamma(A_R \to a+X_I) \ = \ \frac{1}{32\pi k^2}C_3^2\left(\frac{f_a}{f_y}\right)^2\frac{m_{A_R}^3}{f_a^2}. \tag{147}$$

- $\Gamma(A_R \to \psi_{3/2} + \tilde{a})$

  Through an interaction $K = \eta^2\frac{\tilde{c}}{2}(A+A^\dagger)^2X^\dagger X$, we can find

$$\Gamma(A_R \to \psi_{3/2}\tilde{a}) \ \simeq \ \frac{1}{96\pi}\frac{m_{A_R}^5}{m_{3/2}^2 M_{\rm Pl}^2}. \tag{148}$$

- $\Gamma(A_R \to 2\psi_{3/2})$

  Through a mixing between X_R and A_R, we can find

$$\Gamma(A_R \to \psi_{3/2}\psi_{3/2}) \ \simeq \ \frac{1}{48\pi}\left(\frac{U_{12}}{f_a}\right)^2\frac{m_{A_R}^5}{m_{3/2}^2 M_{\rm Pl}^2}. \tag{149}$$

  But this will be suppressed by $\left(\frac{U_{12}}{f_a}\right)^2$, compared to $\Gamma(A_R \to \psi_{3/2}+\tilde{a})$.

- $\Gamma(A_R \to 2g)$

  Through an interaction

$$\frac{\alpha}{8\pi}\frac{A_R}{f_a}F_{\mu\nu}F^{\mu\nu} \tag{150}$$

we obtain

$$\Gamma(A_R \to gg) \simeq \frac{N_g}{16\pi}\left(\frac{\alpha}{4\pi}\right)^2 \frac{m_{A_R}^3}{f_a^2}. \tag{151}$$

For Model 0, we do not have a loop factor in terms of f.

- $\Gamma(A_R \to 2\tilde{a})$

  Through an interaction

$$C_3 m_{\tilde{a}} \frac{A_R}{k f_a} \tilde{a}\tilde{a} \tag{152}$$

in $\frac{f^2 C_3}{3!}(A+A^\dagger)^3$, we will obtain

$$\Gamma(A_R \to \tilde{a}\tilde{a}) \simeq \frac{C_3^2}{4\pi k^2} m_{A_R} \left(\frac{m_{\tilde{a}}}{f_a}\right)^2 \sqrt{1 - 4\frac{m_{\tilde{a}}^2}{m_{A_R}^2}}. \tag{153}$$

Here we used $\partial_{A_R} F^A \simeq 2m_{\tilde{a}}$. Note that we may have also an interaction $\eta^2 \frac{d'}{2k} m_{\tilde{a}} \frac{A_R}{f_a} \tilde{a}\tilde{a}$ via $K\Lambda^{-2} = -\eta^2 \frac{\tilde{c}}{2}(A+A^\dagger)^2 X^\dagger X$, where d' is a coefficient of mixing matrix U_{12}, e.g., $d' = \eta^2(\tilde{d} - \tilde{e})$ for Model 1. But this is at most comparable to the above interaction.

- $\Gamma(A_R \to 2\lambda)$

  Note a decay mode $A_R \to \lambda\lambda$ comes from a gaugino mass interaction $\frac{\alpha}{4\pi} m_{\tilde{a}} \frac{A_R}{f_a} \lambda\lambda$ with using $\partial_{A_R} F^A \simeq 2m_{\tilde{a}}$. Thus

$$\Gamma(A_R \to \lambda\lambda) \simeq N_g \left(\frac{\alpha}{4\pi}\right)^2 \frac{m_{A_R}}{4\pi} \left(\frac{m_{\tilde{a}}}{f_a}\right)^2 \sqrt{1 - 4\frac{m_{\tilde{a}}^2}{m_{A_R}^2}}. \tag{154}$$

is suppressed by $N_g \left(\frac{\alpha}{4\pi}\right)^2$, compared to $\Gamma(A_R \to \tilde{a}\tilde{a})$. Note that $\Gamma(A_R \to \tilde{h}_u \tilde{h}_d)$ would be comparable to this mode, supposing we have $K = c_\mu (A+A^\dagger) H_u H_d + c_B (A+A^\dagger)^2 H_u H_d$, $c_\mu \sim \frac{\alpha}{4\pi}$. For Model 0, $k\frac{\alpha}{4\pi} m_{\tilde{a}}$ should be replaced by $m_{3/2} \sim M_{1/2}$.

- $\Gamma(A_R \to 2\Phi)$

  For a decay mode $A_R \to 2\Phi$, $K = k\left(\frac{\alpha}{4\pi}\right)^2 (A+A^\dagger)^2 \Phi^\dagger \Phi$ is important. This decay can occur via soft scalar mass $\frac{\alpha}{4\pi}\frac{m_{\tilde{a}} m_0}{\sqrt{k} f_a} A_R \Phi^\dagger \Phi$. The amplitude is given by

$$\Gamma(A_R \to 2\Phi) \sim \frac{N_m}{N_g}\left(\frac{m_0}{m_{A_R}}\right)^2 \times \Gamma(A_R \to \lambda\lambda). \tag{155}$$

The decay may also occur via derivative interaction $\left(\frac{\alpha}{4\pi}\right)^2 \langle A_0 \rangle \frac{A_R}{f_a} \Phi^\dagger \partial^2 \Phi$. However, this effect is much smaller than the above result. For Model 0, we will have a similar result, except that an interaction $\frac{A_R}{f} m_0^2 \Phi^\dagger \Phi$ becomes relevant.

- $\Gamma(A_R \to 2X_I)$ in Model 1

  Note that we have $\frac{U_{12}}{\tilde{d}\langle X\rangle} \sim \eta^2$ for Model 1. (Recall that $\langle X\rangle$ is a dimensionful parameter.) Then we have interaction $\frac{U_{12}}{\langle X\rangle}\frac{A_R}{f_a}\left(\partial_\mu X_I \partial^\mu X_I + \delta\partial_\mu X_R \partial^\mu X_R\right)$ for Model 1. Thus, through this interaction we can find

$$\Gamma(A_R \to X_I X_I) \quad \sim \quad \frac{1}{64\pi}\left(\frac{U_{12}}{\langle X\rangle}\right)^2 \frac{m_{A_R}^3}{f_a^2} \;\text{ for Model 1 } X_I. \tag{156}$$

  Here $\frac{U_{12}}{\langle X\rangle} \sim \eta^2$ for Model 1.

D.3 Axino decay

- $\Gamma(\tilde{a} \to \lambda + g)$

  Through an interaction in gauge kinetic term

$$\frac{\alpha}{16\pi f_a}\tilde{a}\sigma^{\mu\nu}\lambda^a F^a_{\mu\nu} + c.c., \tag{157}$$

  the heavy axion mainly decays into gluino and gluon because of the strong interaction. Then we have

$$\Gamma(\tilde{a} \to \lambda + g) \quad \simeq \quad N_g \frac{\alpha^2}{256\pi^3}\frac{m_{\tilde{a}}^3}{f_a^2} \tag{158}$$

  For Model 0, there is no loop factor in terms of f.

- $\Gamma(\tilde{a} \to \psi + \Phi)$

  Here we denoted ψ as the MSSM matter fermions. Note that we also have the fermion-sfermion-axino interaction

$$\frac{\alpha^2}{\sqrt{2}\pi^2}\frac{M_{1/2}}{f_a}\log\left(\frac{f_a}{M_{1/2}}\right)\Phi\psi\tilde{a} \tag{159}$$

  via loop correction by the MSSM gauge interactions [67]. However, this interaction is irrelevant since axino mass is large: amplitude $\Gamma(\tilde{a} \to \psi + \Phi)$ will be suppressed by $\frac{N_m}{N_g}\left(\alpha\log\left(\frac{f_a}{M_{1/2}}\right)\right)^2 (M_{1/2}/m_{\tilde{a}})^2$ compared to $\Gamma(\tilde{a} \to \lambda + g)$.

- $\Gamma(\tilde{a} \to \psi_{3/2} + a)$

  Through an interaction

$$\frac{i}{M_{\rm Pl}}\tilde{a}\sigma^\mu\bar{\sigma}^\nu\psi_\mu\partial_\nu a \tag{160}$$

or equivalently,

$$i\frac{m_{\tilde{a}}}{\sqrt{6}m_{3/2}M_{\rm Pl}}\tilde{a}\sigma^{\mu}\bar{\psi}_X\partial_{\mu}a, \tag{161}$$

this decay will occur. Here the above goldstino interaction originate from $-\eta^2\Lambda^2\frac{\tilde{c}}{2}(A+A^{\dagger})^2|X|^2$ or $\frac{C_3 f^2}{3!}(A+A^{\dagger})^3$. Then we find

$$\Gamma(\tilde{a}\to\psi_{3/2}+a) \simeq \frac{1}{96\pi}\frac{m_{\tilde{a}}^5}{m_{3/2}^2 M_{\rm Pl}^2} \tag{162}$$

- $\Gamma(\tilde{a}\to\psi_{3/2}+X_I)$ in Model 1

  In Model 1, we can have an R-breaking interaction $m_{\tilde{a}}\frac{X_I}{x_0\Lambda}\frac{f_a}{f_y}\tilde{a}\psi_X$ in K_{AX}. Thus we find

$$\Gamma(\tilde{a}\to\psi_{3/2}+X_I) \simeq \frac{1}{8\pi}\left(\frac{f_a}{f_y}\right)^2\frac{m_{\tilde{a}}^3}{(x_0\Lambda)^2}. \tag{163}$$

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